Calc β, BRIDGE 2018 **Name:** _____ **Due:** Thursday, July 26

Homework #14: L'Hospital's rule and anti-differentiation

Note: Your work can only be assessed if it is legible.

1. Evaluate the following limits:

   (a) $\lim_{t \to 0} \dfrac{e^{2t} - 1}{\sin t}$

   (b) $\lim_{x \to 1} \dfrac{x^2 - 1}{x^2 - x}$

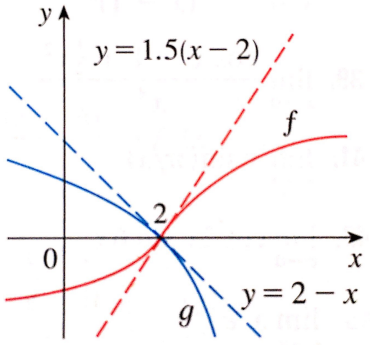
   (c) $\lim_{x \to 0} \dfrac{(x+1)^{13} - 13x - 1}{x^2}$

(d) $\lim_{x \to 0} \dfrac{x - \tan x}{x - \sin x}$

(e) $\lim_{x \to \infty} x^3 e^{-x^2}$

(f) $\lim_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x$

2. Use the graphs of f and g and their tangent lines at $(2, 0)$ to find $\lim_{x \to 2} \dfrac{f(x)}{g(x)}$.



3. Complete the following table.

Function	Particular antiderivative	Function	Particular antiderivative
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sin x$	
$\frac{1}{x} \ (x > 0)$		$\cos x$	$\sin x$
e^x		$\sec^2 x$	
a^x	$\dfrac{a^x}{\ln a}$	$\sec x \tan x$	
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x$	$\dfrac{-1}{\sqrt{1-x^2}}$	
$\dfrac{1}{1+x^2}$			

4. Find the most general antiderivative of the function (use C as any constant).

   (a) $f(x) = 1 + 2x + 3x^2 + 7x^3$

   (b) $f(x) = 2\cos x + e^x - 3\sin x$

   (c) $f(x) = e^2$

5. Find a function $f(x)$ such that

$$f''(x) = 8x^3 + 5, \quad f(0) = 1, \quad f'(1) = 8.$$

6. The fastest recorded speed an F1 car has hit in a race is roughly 231 miles per hour or about 339 ft/s. Suppose you take such a race car onto an unrestricted section of the autobahn[1] to test its brakes. The maximum braking deceleration an F1 car can apply is 6.3 g[2] or about -202 ft/s^2.

   If, at top speed, you apply the brakes constantly at this force to stop the car, how far will you travel before the car comes to a stop?[3]

   Hint: Use feet and seconds in your units.

7. T/F: (with justification) The antiderivative of $\cos(x^2)$ is $\sin(x^2) + C$.

[1] The fastest recorded speed on the autobahn during normal operation (so not for a speed test) is 236 miles per hour.

[2] Here g refers to g-force: 5 g is equivalent to 5 times the acceleration due to gravity on the surface of the earth. For a little context, F1 drivers have reported forced exhalation at 6 g ("having the wind knocked out of you"); 3 g is the maximum acceleration experienced by a Space Shuttle launching or reentering the atmosphere (2.5 - 3 g is what you experience on a carnival "Gravitron"); acrobatic airplane pilots are permitted to force at most 30 g; the Mantis Shrimp (which is a whole footnote on its own) strikes prey with its claw at 10,400 g; and finally 2×10^{11} g is the force of gravity near the surface of a neutron star.

[3] How long would it take you?