Homework # 12: The geometry of a function via its derivatives

Note: Your work can only be assessed if it is legible. A single final answer will be considered a guess and awarded minimal credit.

1. Consider the function $f(x) = 2 + 2x^2 - x^4$.
   (a) Find the critical numbers of $f(x)$.

   (b) On which intervals is $f(x)$ increasing? Decreasing?

   (c) On which intervals is $f(x)$ concave up? Concave down?

(d) Find the exact x-values of the inflection points of $f(x)$.

(e) Sketch $f(x)$.

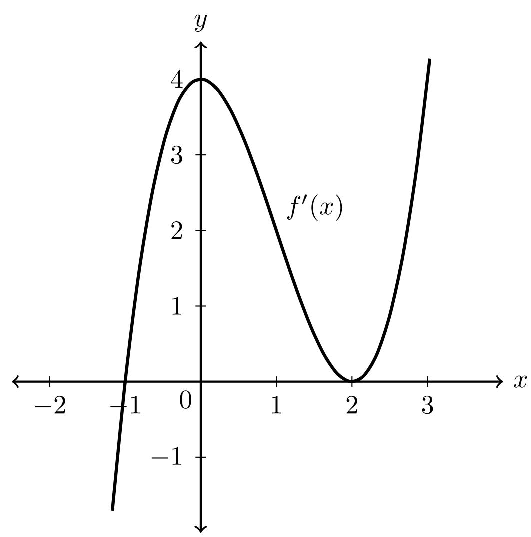
2. For the following functions determine the exact x-values of all local maxima and minima. You must classify each max and min as such and verify your claim in each case via a derivative test.

   (a) $f(x) = x^5 - 2x^3$

   (b) $f(x) = x - 2\sin x$ for $-2\pi < x < 2\pi$.

(c) $f(x) = e^{-x} - e^{-3x}$ for $x > 0$

3. Below is a graph of $y = f'(x)$. Determine the intervals where $f(x)$ is increasing and decreasing, the intervals where $f(x)$ concave up and concave down, the x-values where $f(x)$ has local maxima and minima, and the x-values where $f(x)$ has inflection points.



4. (a) T/F (with justification) If $f(x)$ is a differentiable function on (a, b) and $f'(c) = 0$ for some c in (a, b) then $f(x)$ has a local maximum or minimum value at $x = c$.

(b) T/F (with justification) If a functin $f(x)$ on the interval $(-1, 1)$ is twice differentiable and $f''(c) = 0$ for some c in $(-1, 1)$ then $f(x)$ has an inflection point at $x = c$.