Calc β, BRIDGE 2018 **Name:** _____ **Due:** Monday, July 22

Homework #10: Linearization and related rates

Note: This portion will be done in class.

1. Consider a cube of side length x which expands over time. Give the rate at which the volume of the cube V increases in terms of the rate at which x changes.

2. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast it the radius of the balloon increasing when the diameter is 50 cm?

3. Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = k$, where k is a constant. Suppose that while compressing a certain gas, we have at a particular instant that the volume is 600 cm^3, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant?

4. On a particularly windy day, a park-goer decides to fly a kite. They decide to fly the kite at a constant altitude of 300ft. At this height the wind pulls the kite horizontally away from the kite-flier at a rate of 44ft/sec.[1] How fast will the flier be releasing the string when the kite is 400 ft away (from the park-goer)?

[1] For those curious, this wind speed equates to a 6 of 12 on the Beaufort wind force scale. A 0 on the scale corresponds to essentially no wind while a 12 is hurricane force. A 6 on the scale would mean an umbrella is unpleasant to use.

Note: You are responsible for completing this portion.

5. Find the linearization of $f(x) = x^{3/4}$ at $x = 16$ use it to approximate $(17)^{3/4}$.

   Note: You need not find a decimal answer; if you do, you may be interested in comparing it against the approximation of $(17)^{3/4} \approx 8.37214402859$.

6. (a) Linearize $f(x) = \ln x$ at $x = 1$ and use it to approximate $\ln 1.1$ and $\ln 2$.

   Note: You need not find a decimal answer; if you do, you may be interested in comparing it against the approximation of $\ln 1.1 \approx 0.0953101798043$.

   (b) To make a linear approximation more accurate, we need take the point of tangency closer to the input we desire to approximate.

   Above you approximated $\ln 2$ using a linearization at $x = 1$. Use a linearization of $\ln x$ at $x = e$ to approximate 2.

   Note: You need not find a decimal answer; you may be interested to know $2/e \approx 0.735758882343$ and $\ln 2 \approx 0.69314718056$.

7. (a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

(b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1m/s, how fast is the area of the spill increasing when the radius is 30m?

8. A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 meter higher than the bow of the boat. If the rope is pulled in at a rate of 1 meter per second, how fast is the boat approaching the dock when it is 8 meters from the dock?

9. Gravel is being dumped from a conveyor belt at a rate of 10 ft^3/min. The coarseness is such that the gravel forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 11 ft high?

   Hint: The formula for volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.