Calc β, BRIDGE 2018 **Name:** _____ **Due:** Tuesday, July 17

Homework #8: Implicit differentiation

Note: Your work can only be assessed if it is legible.

1. Find $\dfrac{dy}{dx}$ using implicit differentiation. Solve for $\dfrac{dy}{dx}$ in terms of x and y in each case.

   (a) $2x^3 + x^2y - xy^3 = 2$

   (b) $\cos(xy) = 1 + \sin y$

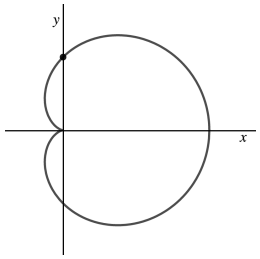
   (c) $e^y \sin x = x + xy$

2. Use implicit differentiation to find an equation of the tangent line to the curve

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

at the point $(0, 1/2)$.

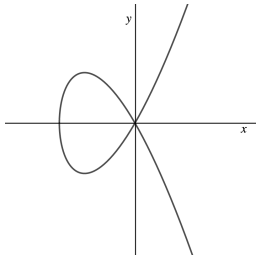
Note: The graph of this equation is known as a cardioid (see below). This is not a graph of a function but we can still geometrically analyze it using implicit differentiation.



3. The curve with equation

$$y^2 = x^3 + 3x^2$$

is called the Tschirnhuasen cubic (see below). At what points does this curve have horizontal tangents?



In class we used implicit differentiation to find derivatives of a couple of the inverse functions in this table.

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$\arcsin x$	$\dfrac{1}{\sqrt{1-x^2}}$	$\ln x$	$\dfrac{1}{x}$
$\arccos x$		$\log_a x$	
$\arctan x$			

Here you will use implicit differentiation to find the rest.

4. *Inverse trig. functions.* Simplify your answers.
   (a) Use implicit differentiation to find the derivative of $y = \arccos x$.

   (b) Use implicit differentiation to find the derivative of $y = \arctan x$.

5. Use implicit differentiation to find the derivative of $y = \log_a x$.

6. In class, we used logarithmic differentiation to show that for any real number n, $(x^n)' = nx^{n-1}$. Use that same technique to find the derivative of the following functions.

   (a) $y = \sqrt{x}^{\,x}$

   (b) $y = x^{\cos x}$

7. Differentiate the following functions. You may use any rule or identity.

   (a) $y = \ln x^2$

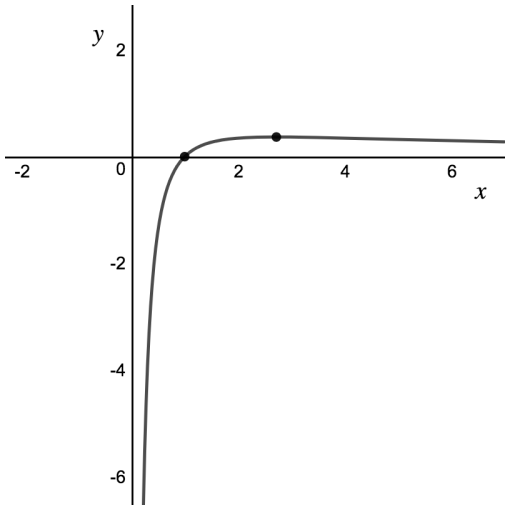
(b) $f(x) = \dfrac{1}{x^3}$

(c) $f(x) = \dfrac{1}{\sqrt[3]{x}}$

(d) $y = \log_2(\arctan x)$

(e) $f(x) = x \ln x - x$

8. Here is a graph of the function $y = \dfrac{\ln x}{x}$.



Find equations of the tangent lines to this graph at:

(a) $x = 1$

(b) and $x = e$.