Calc β, BRIDGE 2018 **Name:** _____ **Due:** Monday, July 16

Homework #7: "Sharpening our tools"

Note: Your work can only be assessed if it is legible.

1. **Basic derivatives.** Give the derivative of each of the following functions. You need not show your work.

   $f(x) = c$, a constant

   $f(x) = x^n$, for a n a positive integer

   $f(x) = \sin x$

   $f(x) = \csc x$

   $f(x) = \cos x$

   $f(x) = \sec x$

   $f(x) = \tan x$

   $f(x) = \cot x$

   $f(x) = e^x$

   $f(x) = a^x$, for $a > 0$

2. **Rules for differentiation.** Let $f(x)$ and $g(x)$ be differentiable functions. State the following rules of differentiation. (I have done the first one for you.)

   (a) State the *sum/difference rule*.

   $$(f(x) \pm g(x))' = f'(x) \pm g'(x).$$

(b) State the *product rule*.

(c) State the *quotient rule*. (Be sure to include the extra assumption for the function in the denominator.)

(d) State the *chain rule*.

3. **Working with the sum and difference rules.** Differentiate each function.

(a) $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.

(b) $f(x) = \cot x - \csc x$.

(c) $f(x) = \sin x + x^5 - e^5$.

(d) $f(x) = (x^5 - x^{1000}) + (5^x - 1000^x)$

4. **Working with the product and quotient rules.** Differentiate the following functions. You need not simplify.

(a) $f(x) = e^x \sin x$

(b) $f(x) = \dfrac{e^x}{\sin x}$

(c) $f(x) = (x^3 + 2x)e^x$

(d) $f(x) = \dfrac{e^x}{x^3 + 2x}$

(e) $f(x) = \dfrac{e^x \sin x}{(x^3 + 2x)e^x}$.

(f) $f(x) = \dfrac{1 - xe^x}{x + e^x}$.

5. **Working with the chain rule.** Differentiate the following functions. If you use a result from a previous question, mention which question and part. You may use the fact that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ without justification.

   (a) $f(x) = (x^4 + 3x^2 - 2)^5$

   (b) $f(x) = \tan(e^3 + x^3)$

   (c) $f(x) = \cos(e^x) + e^{\cos x}$

   (d) $f(x) = 2^{x^2-1}$

   (e) $f(x) = \sqrt{1 - 2x}$

   (f) $f(x) = (e^x + \sin x)^{256}$

6. **Working with multiple rules.** Differentiate the following functions. If you use a result from a previous question, mention which question and part. You may use the fact that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ without justification.

   (a) $f(x) = \sin^2 x + \cos^2 x$.

   (b) $f(x) = \sqrt{9\sin^2 x + 9\cos^2 x}$

   (c) $f(x) = \left(\frac{x^2+1}{x^2-1}\right)^3$

   (d) $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

   (For those that are curious, this function is actually the hyperbolic tangent function.)

   (e) $f(x) = e^{t \sin 2t}$

   (f) $f(x) = \sin(\sin(\sin x))$

7. Given below is a table of values for differentiable functions $f(x)$ and $g(x)$ as well as their derivatives.

x	1	2	3	4
$f(x)$	3	6	6	11
$f'(x)$	1	0	0	1
$g(x)$	1	3	5	4
$g'(x)$	1	2	3	4

(a) If $a(x) = f(x) + 2g(x)$, what is $a'(1)$?

(b) If $b(x) = f(x)g(x)$, what is $b'(2)$?

(c) If $c(x) = \dfrac{f(x)}{g(x)}$, what is $c'(3)$?

(d) If $d(x) = f(g(x))$, what is $d(4)$?

(e) What is $d'(4)$?